VI Iberoamerican Workshop on Orthogonal Polynomials and Applications

VI Encontro Iberoamericano de Polinômios Ortogonais e Aplicações (EIBPOA)

May 9–12, 2017
Uberaba, MG, Brazil

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Event

The Iberoamerican Workshops in Orthogonal Polynomials and Applications (Encuentro Iberoamericano de Polinomios Ortogonales y Aplicaciones - EIBPOA, in Spanish) were created to encourage research in the fields of approximation theory, special functions, orthogonal polynomials and applications among graduate and nongraduate students as well as young researchers from Latin America, Spain and Portugal.

These events are conducted by the research group Iberoamerican Group of Orthogonal Polynomials and Applications (Grupo de Iberoamericano de Polinomios Ortogonales y Aplicaciones - GIBPOA, in Spanish) see https://sites.google.com/site/gibpoa/.

The previous editions of EIBPOA were held in Bogotá, Colombia in 2011, in Colima, Mexico in 2012, in São José do Rio Preto, Brazil in 2013, again in Bogotá in 2014 and lastly Mexico DF, in 2015.

Place

The sixth edition of the EIBPOA will be held in Uberaba, MG, Brazil, from May 9-12, 2017 at the Universidade Federal do Triângulo Mineiro (UFTM), in the auditorium of UFTM - Centro Educacional e Administrativo, which is located at Rua Frei Paulino, number 30, Uberaba, MG.

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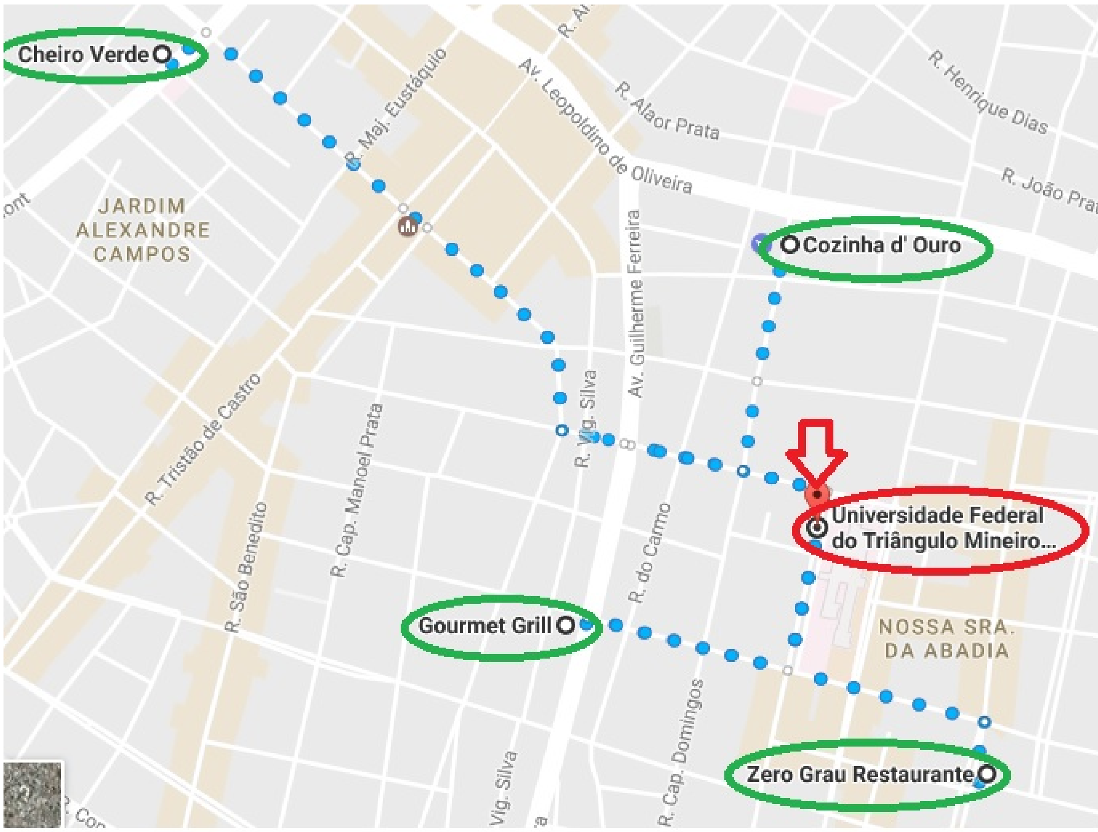

- Restaurante Cozinha d' Ouro is located 550 meters from the UFTM at *Praça Manoel Terra, número 432*, Uberaba, MG (432 Manoel Terra Square). `www.cozinhadouro.com.br`.
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Conference Schedule

	Tuesday 09	Wednesday 10	Thursday 11	Friday 12
08:30–09:00	Registration	Luis E. Garza	Luis E. Garza	Luis E. Garza
09:00–09:30	Opening Session			
09:30–10:00	Opening Session	Coffee break	Coffee break	Coffee break
10:00–11:00	Kerstin Jordaan	Zelia da Rocha	Lidia Fernández	A. Sri Ranga
11:00–11:30	Luis E. Garza	Vanessa Botta	Antonia Delgado	Luana Ribeiro
11:30–12:00		Herbert Dueñas	M. Domínguez	Rafaela Bonfim
12:00–14:00	Lunch time	Lunch time	Lunch time	Lunch time
14:00–15:00	Ana P. Peron	Ana P. Peron	Ana P. Peron	Ana P. Peron
15:00–15:30	Guilherme Silva	Maxim Yattselev	C.F. Bracciali	D.O. Veronese
15:30–16:00			Jairo Silva	Closing Session
16:00–16:30	Coffee break	Coffee break Poster Session	Coffee break	
16:30–17:00	Mario Castro	Erick Delgado	Discussion hour	
17:00–17:30	Jean Guella	Luis Molano		

Schedule by day

Tuesday, May 09th

Time	Activity	Chairman
08:30 – 09:00	Registration	
09:00 – 10:00	Opening Session	
10:00 – 11:00	Kerstin Jordaan *Properties of Some Semi-Classical Orthogonal Polynomials*	A. Sri Ranga
11:00 – 12:00	Luis E. Garza *Spectral Transformations of Orthogonal Polynomials: A Matrix Perspective I*	
12:00 – 14:00	Lunch	
14:00 – 15:00	Ana Paula Peron *Uma Relação entre Polinômios Ortogonais e Funções Positivas Definidas I*	C. Bracciali
15:00 – 16:00	Guilherme L. F da Silva *Critical Measures and Zeros of Polynomials*	
16:00 – 16:30	Coffee Break	
16:30 – 17:00	Mario Henrique de Castro *Super-Exponential Decay for Eigenvalues of Positive Integral Operators on the Spheres*	D. Veronese
17:00 – 17:30	Jean Carlo Guella *A Limit Formula for Semigroups Defined by Fourier-Jacobi Series*	

Wednesday, May 10th

Time	Activity	Chairman
08:30 – 09:30	Luis E. Garza *Spectral Transformations of Orthogonal Polynomials: A Matrix Perspective II*	H. Felix
09:30 – 10:00	Coffee Break	
10:00 – 11:00	Zélia da Rocha *Perturbed Chebyshev Polynomials*	H. Felix
11:00 – 11:30	Vanessa Botta *Three Term Recurrence Relation and BDF Methods*	
11:30 – 12:00	Herbert Dueñas *Sobolev Orthogonal Polynomials on Product Domains in Several Variables*	
12:00 – 14:00	Lunch	
14:00 – 15:00	Ana Paula Peron *Uma Relação entre Polinômios Ortogonais e Funções Positivas Definidas II*	M. Domínguez
15:00 – 16:00	Maxim L. Yattselev *Symmetric Contours and Convergent Interpolation*	
16:00 – 16:30	Poster Session and Coffee Break	
16:30 – 17:00	Erick Manuel Delgado Moya *Orthogonal Polynomials in the Solution of the Problem of Optimal Control*	M. Domínguez
17:00 – 17:30	Luis Alejandro Molano Molano *Asymptotics of Sobolev Orthogonal Polynomials for Hermite (1,1)-Coherent Pairs*	

Thursday, May 11th

Time	Activity	Chairman
08:30 – 09:30	Luis E. Garza *Spectral Transformations of Orthogonal Polynomials: A Matrix Perspective III*	H. Dueñas
09:30 – 10:00	Coffee Break	
10:00 – 11:00	Lidia Fernández *On Orthogonal Polynomials in Several Variables*	H. Dueñas
11:00 – 11:30	Antonia Delgado *Sobolev Orthogonal Polynomials in Several Variable*	
11:30 – 12:00	Manuel Domínguez de la Iglesia *Some Recent Developments about Birth-and-death Models and Orthogonal Polynomials*	
12:00 – 14:00	Lunch	
14:00 – 15:00	Ana Paula Peron *Uma Relação entre Polinômios Ortogonais e Funções Positivas Definidas III*	V. Botta
15:00 – 15:30	Cleonice F. Bracciali *L-orthogonal Polynomials, Toda Lattice and Lax Pairs*	
15:30 – 16:00	Jairo Santos da Silva *Verblunsky Coefficients Related with Periodic Real Sequences and Associated Measures on the Unit Circle*	
16:00 – 17:30	Coffee Break and Discussion hour	

Friday, May 12th

Time	Activity	Chairman
08:30 – 09:30	Luis E. Garza *Spectral Transformations of Orthogonal Polynomials: A Matrix Perspective IV*	V. Botta
09:30 – 10:00	Coffee Break	
10:00 – 11:00	A. Sri Ranga *Para–orthogonal Polynomials on the Unit Circle: Some Recent Developments*	V. Botta
11:00 – 11:30	Luana de Lima Silva Ribeiro *Electrostatic Model for the Zeros of Romanovski Polynomials*	
11:30 – 12:00	Rafaela Neves Bonfim *Strict Positive Definiteness of Product Covariance Functions on Manifolds*	
12:00 – 14:00	Lunch	
14:00 – 15:00	Ana Paula Peron *Uma Relação entre Polinômios Ortogonais e Funções Positivas Definidas IV*	L. Garza
15:00 – 15:30	Daniel O. Veronese *Orthogonal Polynomials on the Real Line Generated by a Perturbation of Symmetric Orthogonal Polynomials*	
15:30 – 16:00	Closing Session	

Abstracts

Short Courses

Spectral transformations of orthogonal polynomials: A matrix perspective

Luis E. Garza
Universidad de Colima, Mexico

Two important matrices appear frequently in the theory of orthogonal polynomials on the real line: a tridiagonal matrix (that represents the multiplication by x operator in terms of the basis of orthogonal polynomials) and a Hankel matrix that contains the moments associated with the orthogonality measure. For orthogonal polynomials on the unit circle, we have analogously the GGT (or CMV) matrix for the multiplication operator, and the matrix of moments, which in this case has Toeplitz structure. These matrices characterize the existence of families of orthogonal polynomials, and their analysis yield algebraic and analytic properties of such polynomials. On the other hand, in the last decades, some attention has been paid to the so-called canonical spectral transformations (i.e. Christoffel, Uvarov and Geronimus) of orthogonal polynomials. In this course, we will show that these transformations can be expressed as certain factorizations of the matrices mentioned above. Furthermore, we will present an alternative matrix approach that has appeared recently in the literature, which allows to characterize the orthogonality in terms of lower triangular infinite matrices. Some open problems will be posed.

Uma relação entre polinômios ortogonais e funções positivas definidas

Ana Paula Peron
Universidade de São Paulo, Brazil

Polinômios ortogonais e funções positivas definidas têm sido temas relevantes em várias linhas de pesquisa nas últimas décadas. Neste minicurso apresentaremos uma breve introdução à teoria de harmônicos esféricos. Introduziremos os polinômios de Legendre que desempenham um papel fundamental no estudo dos harmônicos esféricos. Como uma extensão dos polinômios de Legendre, os polinômios de Gegenbauer serão também apresentados. Ambos são importantes exemplos de polinômios ortogonais. Ainda, os polinômios de Gegenbauer são uma ferramenta fundamental no estudo de funções positivas definidas em esferas reais d-dimensionais. Tais funções estão relacionadas à solução de problemas de interpolação, de aproximação e estatísticos. Será apresentado o conceito de positividade definida, exemplos e algumas propriedades. Pretendemos, neste mini-curso, explanar sobre alguns dos principais resultados neste amplo tema, dando dicas sobre algumas linhas de pesquisa atuais.

Conferences

On orthogonal polynomials in several variables

LIDIA FERNÁNDEZ
UNIVERSIDAD DE GRANADA, SPAIN

The aim of this talk is to present some general aspects related with orthogonal polynomials in several variables. Mainly, I will talk about orthogonality on the ball and on the simplex because they are, in a sense, the most appealing examples. I will focus on the construction of the basis for the classical inner products and other interesting modifications. Some of the algebraic and analytical properties of these polynomials will be showed. Moreover, the study of this basis is going to be related with some engaging properties about the zeros of this polynomials in the two variables case.

REFERENCES

[1] C. F. DUNKL, Y. XU, *Orthogonal polynomials of several variables*, Encyclopedia of Mathematics and its Applications 81, Cambridge University Press, 2001.

[2] T. H. KOORNWINDER, Two variable analogues of the classical orthogonal polynomials, *Theory and Application of Special Functions*, R. Askey Editor, Academic Press (1975), 435–495. 435-495.

Properties of some semi-classical orthogonal polynomials

KERSTIN JORDAAN
UNIVERSITY OF SOUTH AFRICA, SOUTH AFRICA

Let μ be a positive measure defined on the Borel subsets of the real line for which all the moments

$$\mu_n = \int x^n d\mu(x)$$

exist. Since the Hilbert space $L^2(\mu)$ contains the set of polynomials, Gram-Schmid orthogonalisation can be applied to the set $\{1, x, x^2, \ldots\}$ to obtain a sequence of $\{P_n\}_n \in \mathbb{N}$ of polynomials such that

$$\int P_n(x)P_k(x)d\mu(x) = \delta_{n,k} \qquad n, m = 0, 1, 2, \ldots$$

where $\delta_{n,k}$ is the Kronecker delta and the integration is over the support $S \subset \mathbb{R}$ of the measure μ. The orthogonal polynomials $\{P_n\}$ satisfy a three term recurrence relation of the form

$$xP_n(x) = a_{n+1}P_{n+1}(x) + b_nP_n(x) + a_nP_{n-1}(x)$$

Other characteristic properties of the sequence $\{P_n\}$ are determined by the nature of the measure μ. Given an orthogonality measure, extracting this information from the measure is one of the interesting (and challenging) problems in the study of systems of orthogonal polynomials. We discuss some aspects of this so-called direct problem of orthogonal polynomials for measures which are semi-classical perturbations of classical orthogonality measures. We use the moments together with a connection between orthogonal polynomials and Painlevé equations to obtain explicit expressions for the recurrence coefficients of polynomials associated with a semi-classical Laguerre and a generalized Freud weight. We derive a second-order linear ordinary differential equation and a differential-difference equation satisfied by the generalized Freud polynomials and analyze the asymptotic behavior of the polynomials in two different contexts.

Para–orthogonal polynomials on the unit circle: Some recent developments

A. Sri Ranga

UNESP – Universidade Estadual Paulista, São José do Rio Preto, Brazil

Orthogonal polynomials on the real line always satisfy a three term recurrence relation. Studying the properties of these orthogonal polynomials and also the associated measure from the coefficients of the three term recurrence relation is known to be an interesting topic of research. On the other hand, orthogonal polynomials on the unit circle, in general, do no satisfy a three term recurrence relation. However, we can choose sequences of para-orthogonal polynomials such that they satisfy a nice three term recurrence relation. It turns out one can also study the orthogonal polynomials and the associated measure from the coefficients of such three term recurrence relations.

Perturbed Chebyshev polynomials

Zélia da Rocha

Universidade do Porto, Portugal

In some applications one is led to consider perturbations of orthogonal polynomials translated by a modification on the first coefficients of the second order linear recurrence relation satisfied by these polynomials. This transformation can promote a deep change of properties; nevertheless there is a large set of forms that are preserved by perturbation: the second degree forms. Moreover, second degree forms are also semi–classical. Among the classical forms only certain Jacobi forms are of second degree, including the four Chebyshev forms. Thus, it is important to clarify and explicit the properties of perturbed Chebyshev polynomials. From the point of view of perturbation, the Chebyshev form of second kind is the most simple and the other three forms of Chebyshev can be considered, in a natural way, as perturbed of it.

By means of a general method and the corresponding symbolic algorithm *PSDF* [2,3] based on Stieltjes equations, we are able to explicit several semi–classical properties of perturbed second degree forms namely: the Stieltjes function, the Stieltjes equation, the functional equation, the class, a structure relation and the second order linear differential equation as well as the first moments and the generating function of perturbed forms. Applying the algorithm *PSDF* to the Chebyshev form of second kind, we achieve to explicit the above mentioned properties for perturbations of several fixed orders [2,3] generalizing existent results in literature. From these properties, we can easily derive similar ones for the other three forms of Chebyshev [2].

Also, we consider the problem of finding the connection coefficients [4, 5] that allow to write the perturbed of the Chebyshev polynomials of second kind in terms of the original sequence, and in terms of the canonical basis. Starting from some symbolic results produced by the software *CCOP* [6,7], we achieve to generalize know formulas [1] for any order of perturbation. From the connection relations obtained, we deduce some results about zeros and interception points of these perturbed polynomials.

References

[1] Z. da Rocha, On connection coefficients of some perturbed of arbitrary order of the Chebyshev polynomials of second kind, submitted, (2017).

[2] Z. da Rocha, On the second order differential equation satisfied by perturbed Chebyshev polynomials, J. Math. Anal., 7(1) (2016) 53-69.

[3] Z. da Rocha, A general method for deriving some semi-classical properties of perturbed second degree forms: the case of the Chebyshev form of second kind, J.Comput. Appl. Math., 296 (2016) 677-689.

[4] P. Maroni, Z. da Rocha, Connection coefficients for orthogonal polynomials: symbolic computations, verifications and demonstrations in the *Mathematica* language, Numer. Algor., 63-3 (2013) 507-520.

[5] P. Maroni, Z. da Rocha, Connection coefficients between orthogonal polynomials and the canonical sequence: an approach based on symbolic computation, Numer. Algorithms, 47-3 (2008) 291-314.

[6] P. Maroni, Z. da Rocha, Software CCOP - *Connection Coefficients for Orthogonal Polynomials*, Numer. Algor., (2013), *http://www.netlib.org/numeralgo/*, na34 package.

[7] P. Maroni, Z. da Rocha, Software CCOP - TUTORIAL. Numer. Algor., 40 p. (2013), *http://www.netlib.org/numeralgo/*, na34 package.

Critical measures and zeros of polynomials

Guilherme L. F. da Silva
University of Michigan, USA

The theory of multiple orthogonal (or Hermite-Padé) polynomials has gained a lot of attention in the past few years, partially due to its connection to several models of mathematical physics, such as random matrices, non-intersecting paths and planar growth, to mention a few. In contrast to the standard orthogonality, the behavior of the zeros of large degree multiple orthogonal polynomials is not completely understood, with results available only for somewhat extremal cases (as Nikishin or Angelesco systems) or under strong symmetry assumptions for the potentials.

Motivated towards understanding the general picture, in this talk we plan to discuss the notion of *vector critical measures* for a vector logarithmic energy containing both repulsive and attractive interactions. We obtain several structural results for vector critical measures, in particular showing that they are intimately connected with quadratic differentials on compact Riemann surfaces. We illustrate this connection constructing a family of vector critical measures for cubic potentials, and prove that this family describes the limiting zero distribution for certain multiple orthogonal polynomials.

This is a joint work with Andrei Martínez-Finkelshtein (Universidad de Almería - Spain).

Symmetric contours and convergent interpolation

Maxim L. Yattselev
Indiana University - Purdue University Indianapolis, Indianapolis, USA

The essence of Stahl-Gonchar-Rakhmanov theory of symmetric contours as applied to the multipoint Padé approximants is the fact that given a germ of an algebraic function and a sequence of rational interpolants with free poles of the germ, if there exists a contour that is "symmetric" with respect to the interpolation scheme, does not separate the plane, and in the complement of which the germ has a single-valued continuation with non-identically zero jump across the contour, then the interpolants converge to that continuation in logarithmic capacity in the complement of the contour. The existence of such a contour is not guaranteed. I will discuss how to construct a class of pairs interpolation scheme/symmetric contour with the help of hyperelliptic Riemann surfaces. I will further explain how to obtain formulae of strong asymptotics for the error of interpolation when rational interpolants with free poles of Cauchy transforms of non-vanishing smooth complex densities on such contours are considered.

Strict positive definiteness of product covariance functions on manifolds

Rafaela Neves Bonfim
Universidade de São Paulo, São Carlos, Brazil

Let X denote a compact two-point homogeneous space. Given two continuous and isotropic positive definite kernels on X, we determine necessary and sufficient conditions in order that their product be strictly positive definite on X. We also consider the very same question in the space-time setting, that is, the case $X = G \times S^d$, in which G is a locally compact group, S^d is the unit sphere in $\mathbb{R}^{d+1}$ and both kernels are continuous and isotropic with respect to the component S^d.

(Joint work with Valdir A. Menegatto)

L-orthogonal polynomials, Toda Lattice and Lax pairs

Cleonice Fátima Bracciali
UNESP – Universidade Estadual Paulista, São José do Rio Preto, Brazil

The coefficients of the recurrence relation of orthogonal polynomials, when the measure varies parametrically in a certain way, satisfy the so-called Toda lattice. Here we show that the coefficients of the recurrence relation of L-orthogonal polynomials and kernel polynomials on the unit circle satisfy what we call extended relativistic Toda lattice. A Lax pair for the extended relativistic Toda lattice associated with the recursion coefficients of L-orthogonal polynomials is also established.

(Joint work with J.S. Silva and A. Sri Ranga)

Super-exponential decay for eigenvalues of positive integral operators on the sphere

Mario Henrique de Castro
Universidade Federal de Uberlândia, Brazil

In this work we obtain optimal super-exponential decay rates for eigenvalue sequences of positive integral operators acting on functions defined on the usual m-dimensional unit sphere, under smoothness assumptions for the generating kernels defined by Laplace-Beltrami differentiability. Orthogonal polynomials play an important role in demonstrations via spherical harmonic properties. The results also apply for two-point homogeneous spaces.

(Joint work with Thaís Jordão)

This work partially supported by FAPEMIG, grant # CEX-APQ-00474-14, and FAPESP, grant # 2016/02847-9.

Sobolev orthogonal polynomials in several variables

Antonia Delgado
Universidad de Granada, Spain

Sobolev orthogonal polynomials are those who are orthogonal with respect to an inner product that involves not only the functions, but also their derivatives. In contrast with the one variable case, the study of such polynomials in several variables is a quite recent task, and just a few special examples have been studied. This time we deal with a Sobolev inner product on the unit ball which is defined via the outward normal derivative on the sphere. We will find explicit representation for the corresponding Sobolev orthogonal polynomials and reproducing kernels in terms of classical polynomials on the ball. Also, some algebraic properties and the asymptotic behaviour of Christoffel functions will be deduced.

Sobolev orthogonal polynomials on product domains in several variables

Herbert Dueñas
Institution Universidad Nacional de Colombia, Colombia

We consider the inner product

$$\langle f, g\rangle_S = c\int_{a_2}^{b_2}\int_{a_1}^{b_1} \nabla^2 f(x,y)\cdot\nabla^2 g(x,y) w_1(x) w_2(y)dxdy + \lambda f(c_1,c_2)g(c_1,c_2),$$

where $\lambda > 0$, (c_1, c_2) is some fixer point in $[a_1, b_1] \times [a_2, b_2]$, w_i is a weight function on $[a_i, b_i]$, $i = 1, 2$, and $c = 1/\int_{a_2}^{b_2}\int_{a_1}^{b_1} w_1(x)w_2(y)dxdy$.

Using the ideas presented in [1], we construct the polynomials of several variables which are orthogonal with respect to such inner product for certain weight functions. And as example, the Laguerre product and the Gegenbauer product weight functions are studied.

(Joint work with Natalia Pinzón-Cortés and Omar Salazar Morales)

References

[1] L. Fernández, F. Marcellán, T. Pérez, M. Piñar and Y. Xu, *Sobolev orthogonal polynomials on product domains*, Jour. Comp. App. Math. Vol. 284, N° 15, (2015), 202-215.

Some recent developments about birth-and-death models and orthogonal polynomials

Manuel Domínguez de la Iglesia
Universidad Nacional Autónoma de México, Mexico

I will present some recent results of the spectral analysis of birth-and-death models (classical and bivariate) in connection with orthogonal polynomials.

A limit formula for semigroups defined by Fourier-Jacobi series

Jean Carlo Guella

Universidade de São Paulo, São Carlos, Brazil

I. J. Schoenberg showed the following result in his celebrated paper [Schoenberg, I. J., Positive definite functions on spheres. Duke Math. J. **9** (1942), 96-108]: let $\cdot$ and S^d denote the usual inner product and the unit sphere in $\mathbb{R}^{d+1}$, respectively. If $\mathcal{F}^d$ stands for the class of real continuous functions f with domain $[-1,1]$ defining positive definite kernels $(x,y) \in S^d \times S^d \to f(x \cdot y)$, then the class $\cap_{d\geq 1}\mathcal{F}^d$ coincides with the class of power series oc radius at least 1 and center on 0, with nonnegative coefficients. We present an extension of this result to some classes of continuous functions defined by Fourier-Jacobi expansions with nonnegative coefficients.

Let $\mathcal{F}^{\alpha,\beta}$ denote the family of all real continuous functions on $[-1,1]$ that possess a series representation in the form

$$\sum_{k=0}^{\infty} a_k^{\alpha,\beta} P_k^{(\alpha,\beta)}(t), \quad t \in [-1,1],$$

in which all the coefficients $a_k^{\alpha,\beta}$ are nonnegative and the series is convergent at $t = 1$. Our main theorem is: Let $\{\alpha_m\}$ and $\{\beta_m\}$ be sequences in $[-1/2, \infty)$ with $\{\alpha_m\} \to \infty$, $\{\beta_m \alpha_m^{-1}\} \to 0$ and $\alpha_m \geq \beta_m$. A continuous function $f : [-1,1] \to \mathbb{R}$ belongs to $\cap_{m=1}^{\infty} \mathcal{F}^{\alpha_m,\beta_m}$ if, and only if, f has the representation

$$f(t) = \sum_{n=0}^{\infty} a_n \left(\frac{1+t}{2}\right)^n, \quad t \in [-1,1],$$

in which all the a_n are nonnegative and $\sum_{n=0}^{\infty} a_n < \infty$. With this result, we establish a version of Schoenberg theorem in the case in which the spheres S^d are replaced with compact two-point homogeneous spaces.

Asymptotics of Sobolev orthogonal polynomials for Hermite (1,1)-coherent pairs

Luis Alejandro Molano Molano

Universidad Pedagógica y Tecnológica de Colombia, Duitama, Colombia

In this talk we will discuss asymptotics of the monic polynomials $\left\{S_n^{\lambda}\right\}_{n\geq 0}$ orthogonal with respect to Sobolev inner product

$$\langle p, q \rangle_S = \int_{\mathbb{R}} p(x)q(x)d\mu_0 + \lambda \int_{\mathbb{R}} p'(x)q'(x)d\mu_1,$$

where $\lambda > 0$, $d\mu_0 = e^{-x^2}dx$ and $d\mu_1 = \frac{x^2+a}{x^2+b}e^{-x^2}dx$, with $a, b \in \mathbb{R}^+,\ a \neq b$. It is well known that (μ_0, μ_1) is a pair of symmetric $(1,1)-$coherent measures, this means that there exist sequences

$\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$, with $a_n \neq b_n$ for every $n \in \mathbb{N}$, such that the algebraic relation

$$H_n(x) + b_{n-2}H_{n-2}(x) = Q_n(x) + a_{n-2}Q_{n-2}(x), \tag{1}$$

is satisfied. In (1), $\{Q_n\}_{n\geq 0}$ is the SMOP associated to μ_1 and $\{H_n\}_{n\geq 0}$ is the sequence of monic Hermite polynomials. In this way, it is possible to establish an algebraic relation between the polynomials $\{H_n\}_{n\geq 0}$ and $\{S_n^\lambda\}_{n\geq 0}$, namely

$$S_{n+3}^\lambda(x) + \eta_n(\lambda)S_{n+1}^\lambda(x) = H_{n+3}(x) + \frac{n+3}{n+1}b_nH_{n+1}(x).$$

We will show the behaviour of the sequences $\{a_n\}$, $\{b_n\}$ and $\{\eta_n(\lambda)\}$ when $n \to \infty$, we will study relative asymptotics for Sobolev scaled polynomial and we will obtain Mehler-Heine type formulas.

(Joint work with F. Marcellán and H. Dueñas)

Orthogonal polynomials in the solution of the problem of optimal control

Erick Manuel Delgado Moya

Pontifícia Universidade Católica do Rio de Janeiro, Brazil

An alternative method based on the orthogonal polynomials and their operational properties as the ope-rational matrix of integration, product and coefficient to give a solution to the problem of getting best control is presented in this paper. The approximate numeric integration is omitted by the obtainment of a less expensive computering direct method looking for a better precision.

Three term recurrence relation and BDF methods

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In this work we will show that the characteristic polynomials of the BDF methods (Backward Difference Formulas) satisfy a three term recurrence relation. From the theory of recurrence relation we will explore the behavior of the zeros of the characteristic polynomials of BDF methods, that are the most widely used for the solution of stiff differential equations.

Electrostatic model for the zeros of Romanovski polynomials

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The Romanovski polynomials $R_n^{(\alpha,\beta)}(x)$ are polynomial solutions of the differential equation

$$(1+x^2)Y''(x) + (2\beta x + \alpha)Y'(x) - n(2\beta + n - 1)Y(x) = 0,$$

where $\alpha, \beta \in \mathbb{R}$. These polynomials first appeared in a work of Routh in 1884 and were rediscovered in 1929 by Romanovski in the context of the theory of probability and statistics. Although, in the same way as for the classical orthogonal polynomials, the polynomials $R_n^{(\alpha,\beta)}(x)$ can be defined via a Rodrigues formula, they do not have the standard orthogonal property. However, for $m \neq n$ and for $m + n < 1 - 2\beta$

$$\int_{-\infty}^{\infty} R_n^{(\alpha,\beta)}(x) R_m^{(\alpha,\beta)}(x)(1 + x^2)^{\beta-1} e^{\alpha \arctan(x)} dx = 0.$$

Thus, when $n < -\beta$ the polynomials $R_n^{(\alpha,\beta)}(x)$ satisfy a finite orthogonality relation. In this work we will study a subclass of Romanovski polynomials, where the parameter β also depends on n, and we will give an electrostatic interpretation for their zeros.

(Joint work with A. Martínez-Finkelshtein, A. Sri Ranga and M. Tyaglov)

Verblunsky coefficients related with periodic real sequences and associated measures on the unit circle

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It was shown recently that associated with a pair of real sequences (where one of them is a positive chain sequence) there exists a unique nontrivial probability measure supported on the unit circle. In the present work, our main contribution is to study the behavior of these measures when we impose some restrictions of sign and periodicity on these sequences. Precisely, when this pair is such that the minimal parameter sequence of the positive chain sequence and the other sequence are periodic, we show that the study of these measures is completely equivalent to the study of measures associated with periodic Verblunsky coefficients: which allows us, in this case, to present, to study and to characterize a new space of measures on the unit circle

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Orthogonal polynomials on the real line generated by a perturbation of symmetric orthogonal polynomials

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In this work we show how is possible to obtain sequences of orthogonal polynomials on the real line starting from a given sequence of functions which can be considered as a perturbation of symmetric orthogonal polynomials. We also study some connections with the unit circle and related results.

Posters

Teoremas clássicos de Sturm-Liouville para polinômios de Gegenbauer

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Os teoremas clássicos serão aplicados para a análise de crescimento e desenvolvimento de certas funções que envolvem os zeros dos polinômios de Gegenbauer.

Optimal quadratures with preassigned multiplicities of nodes

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Let W be a given class of sufficiently smooth functions defined on the interval (α, β) and $[a, b] \subset (\alpha, \beta)$. A quadrature formulae in W is of the form $I(f) := \int_a^b f(x)dx \approx \sum_{i=1}^{k} \sum_{j=0}^{\mu_i - 1} a_{ij} f^{(j)}(x_i) := S(a, x; f)$ with some real coefficients $\mathbf{a} = (a_{ij})$ and n distinct nodes $x_1, \ldots, x_n$ in (α, β) with multiplicities $\nu_1, \ldots, \nu_n$ respectively. A quadrature with coefficients $\mathbf{c}$ is said optimal for the nodes $x_1, \ldots, x_n$ if $\sup_{f \in W} |I(f) - S(\mathbf{c}, x, f)| = \inf_{\mathbf{a}} \sup_{f \in W} |I(f) - S(\mathbf{a}, x, f)| := R(x)$. Let the natural numbers $\nu_1, \ldots, \nu_n$ fixed. The nodes that minimizes $R(x)$ are called optimal nodes. The existence of optimal nodes and its respective optimal quadrature will be demonstrated in a maximal subspace of polynomial (Hermite quadrature) and for a class of smooth functions.

(Joint work with Vanessa G.P. Ferraz)

References

[1] B. Bojanov, On the existence of optimal quadrature formulas for smooth functions, Calcolo Vol, XVI, fasc. I, (1979), 61-70.

[2] B. Bojanov, Optimal recovery of functions and integrals. *In: First European Congress of Mathematics*. Birkhäuser Basel, 1994. p. 371-390.

Zeros of polynomials that satisfy a special R_{II} type recurrence relation

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We will discuss numerical methods to find the zeros of polynomials that belong to a sequence of polynomials $\{P_n\}_{n\geq 0}$ and satisfy the recurrence relation

$$P_{n+1}(x) = (x - c_{n+1})P_n(x) - d_{n+1}(x^2 + 1)P_{n-1}(x), \quad n \geq 1,$$

with $P_0(x) = 1$ e $P_1(x) = x - c_1$. This recurrence relation is known as R_{II} type recurrence (Ismail, Masson 1995). If $\{c_n\}_{n\geq 1}$ is a real sequence and $\{d_{n+1}\}_{n\geq 1}$ is a positive chain sequence, the zeros of these polynomials are real and distinct. It was also proved that the zeros of P_n are solution of a generalized eigenvalue problem, $A_n u = x B_n u$, $n \geq 1$ where A_n is a tridiagonal hermitian matrix, B_n is a symmetric matrix and $u \in \mathbb{R}^n$ (Ismail, Sri Ranga 2016 in arxiv). We will discuss the use of Laguerre's Iteration and QZ-method (Wilkinson 1965 and Golub, Van Loan 2013) to find the zeros as eigenvalues of $A_n u = x B_n u$.

(Joint work with C.F. Bracciali and A. Sri Ranga)

On the zeros of self-inversive polynomials

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In this work we present some classical results about the behavior of the zeros of self-inversive polynomials, as the Cohn's Theorem, for example, related with the number of zeros of a self-inversive polynomials located in the unit circle and the zeros of its derivative. The zeros of self-inversive polynomials are symmetric in the unit circle. Furthermore, we illustrate this study from numerical examples using the software Mathematica 10.

A study about zeros of trinomials

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In this work we analyse the behavior of the zeros of trinomial equations represented by $P(z) = z^n - az^k - 1$, where $a \in \mathbb{C}$. Considering $\sigma(n;k) = \frac{n}{n-k}\left(\frac{n-k}{k}\right)^{\frac{k}{n}}$, following [1] we study the regions of the complex plane where the zeros of $P(z)$ are located, according the conditions $|a| > \sigma(n;k)$ or $|a| < \sigma(n;k)$.

References

[1] A Melman. Geometry of trinomials. Pac. J. of Math, 259 (1), 2012.
